non-terminating non-recurring lying between them. Of course, you can find infinitely many such numbers.

An example of such a number is 0.150150015000150000...

EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

   (i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$

   (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$ (vi) $\frac{329}{400}$

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

   [**Hint :** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

   (i) $0.\bar{6}$ (ii) $0.4\bar{7}$ (iii) $0.\overline{001}$

4. Express 0.99999 .... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

9. Classify the following numbers as rational or irrational :

   (i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

   (iv) 7.478478... (v) 1.101001000100001...

EXERCISE 1.5

1. Classify the following numbers as rational or irrational:

   (i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

   (iv) $\frac{1}{\sqrt{2}}$ (v) 2π

2. Simplify each of the following expressions:

   (i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

   (iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

4. Represent $\sqrt{9.3}$ on the number line.

5. Rationalise the denominators of the following:

   (i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

   (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

1.6 Laws of Exponents for Real Numbers

Do you remember how to simplify the following?

(i) $17^2 \cdot 17^5 =$ (ii) $(5^2)^7 =$

(iii) $\frac{23^{10}}{23^7} =$ (iv) $7^3 \cdot 9^3 =$

Did you get these answers? They are as follows:

(i) $17^2 \cdot 17^5 = 17^7$ (ii) $(5^2)^7 = 5^{14}$

(iii) $\frac{23^{10}}{23^7} = 23^3$ (iv) $7^3 \cdot 9^3 = 63^3$

Example 24 : Factorise $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Solution : The given expression can be written as

$$(2x)^3 + (3y)^3 + 3(4x^2)(3y) + 3(2x)(9y^2)$$
$$= (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2$$
$$= (2x + 3y)^3 \quad \text{(Using Identity VI)}$$
$$= (2x + 3y)(2x + 3y)(2x + 3y)$$

Now consider $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

On expanding, we get the product as

$$x(x^2 + y^2 + z^2 - xy - yz - zx) + y(x^2 + y^2 + z^2 - xy - yz - zx)$$
$$+ z(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + xy^2 + xz^2 - x^2y - xyz - zx^2 + x^2y$$
$$+ y^3 + yz^2 - xy^2 - y^2z - xyz + x^2z + y^2z + z^3 - xyz - yz^2 - xz^2$$
$$= x^3 + y^3 + z^3 - 3xyz \quad \text{(On simplification)}$$

So, we obtain the following identity:

Identity VIII : $\mathbf{x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)}$

Example 25 : Factorise : $8x^3 + y^3 + 27z^3 - 18xyz$

Solution : Here, we have

$$8x^3 + y^3 + 27z^3 - 18xyz$$
$$= (2x)^3 + y^3 + (3z)^3 - 3(2x)(y)(3z)$$
$$= (2x + y + 3z)[(2x)^2 + y^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)]$$
$$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$$

EXERCISE 2.5

1. Use suitable identities to find the following products:
   (i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$ (iii) $(3x+4)(3x-5)$
   (iv) $(y^2+\frac{3}{2})(y^2-\frac{3}{2})$ (v) $(3-2x)(3+2x)$
2. Evaluate the following products without multiplying directly:
   (i) 103×107 (ii) 95×96 (iii) 104×96
3. Factorise the following using appropriate identities:
   (i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$ (iii) $x^2 - \frac{y^2}{100}$

4. Expand each of the following, using suitable identities:
   (i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$ (iii) $(-2x+3y+2z)^2$
   (iv) $(3a-7b-c)^2$ (v) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

5. Factorise:
   (i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$
   (ii) $2x^2+y^2+8z^2-2\sqrt{2}\,xy+4\sqrt{2}\,yz-8xz$

6. Write the following cubes in expanded form:
   (i) $(2x+1)^3$ (ii) $(2a-3b)^3$ (iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$

7. Evaluate the following using suitable identities:
   (i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

8. Factorise each of the following:
   (i) $8a^3+b^3+12a^2b+6ab^2$ (ii) $8a^3-b^3-12a^2b+6ab^2$
   (iii) $27-125a^3-135a+225a^2$ (iv) $64a^3-27b^3-144a^2b+108ab^2$
   (v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

9. Verify : (i) $x^3+y^3=(x+y)(x^2-xy+y^2)$ (ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

10. Factorise each of the following:
    (i) $27y^3+125z^3$ (ii) $64m^3-343n^3$

    [**Hint** : See Question 9.]

11. Factorise : $27x^3+y^3+z^3-9xyz$

12. Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

13. If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$.

14. Without actually calculating the cubes, find the value of each of the following:
    (i) $(-12)^3+(7)^3+(5)^3$
    (ii) $(28)^3+(-15)^3+(-13)^3$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i)	(ii)
Area : $25a^2-35a+12$	Area : $35y^2+13y-12$