## WORKING RULES FOR SOLVING PROBLEMS

**Rule I**

(i) $\sin^{-1}(\sin\theta) = \theta$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta$ if $\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta$ if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\cot^{-1}(\cot\theta) = \theta$ if $\theta \in (0, \pi)$

(v) $\sec^{-1}(\sec\theta) = \theta$ if $\theta \in [0, \pi] - \{\pi/2\}$

(vi) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

**Rule II**

(i) $\sin(\sin^{-1}x) = x$ if $x \in [-1, 1]$

(ii) $\cos(\cos^{-1}x) = x$ if $x \in [-1, 1]$

(iii) $\tan(\tan^{-1}x) = x$ if $x \in \mathbf{R}$

(iv) $\cot(\cot^{-1}x) = x$ if $x \in \mathbf{R}$

(v) $\sec(\sec^{-1}x) = x$ if $x \in \mathbf{R} - (-1, 1)$

(vi) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$ if $x \in \mathbf{R} - (-1, 1)$

**Rule III**

(i) $\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$ if $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$ if $x \in \mathbf{R}$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$ if $x \in \mathbf{R}$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$ if $x \in \mathbf{R} - (-1, 1)$

(vi) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$ if $x \in \mathbf{R} - (-1, 1)$

**Rule IV**

(i) $\operatorname{cosec}^{-1}x = \sin^{-1}(1/x)$ if $x \in \mathbf{R} - (-1, 1)$

(ii) $\sec^{-1}x = \cos^{-1}(1/x)$ if $x \in \mathbf{R} - (-1, 1)$

(iii) $\cot^{-1}x = \begin{cases} \pi + \tan^{-1}(1/x) & \text{if } x < 0 \\ \tan^{-1}(1/x) & \text{if } x > 0 \end{cases}$

**Rule V**

(i) $\sin^{-1}x + \cos^{-1}x = \pi/2$ if $x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \pi/2$ if $x \in \mathbf{R}$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$ if $x \in \mathbf{R} - (-1, 1)$

**Rule VI**

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$ if $xy < 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1+xy}$ if $xy > -1$

**Rule VII**

(i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$
if either $x^2 + y^2 \le 1$ or $xy < 0$.

(ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$
if either $x^2 + y^2 \le 1$ or $xy > 0$

**Rule VIII**

(i) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$ if $x + y \ge 0$

(ii) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$ if $x \le y$

**Rule IX**

(i) $\sin^{-1}\dfrac{2x}{1+x^2} = 2\tan^{-1}x$ if $x \in [-1, 1]$

## VERY SHORT ANSWER TYPE QUESTIONS

1. Find the sum of the following matrices:

(i) $\begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$

(ii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix}$ and $\begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix}$ and $\begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

(iv) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix}$ and $\begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

2. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find $A - B$.

3. Find A + B if defined, for the following matrices:

(i) $A = [4 \quad 3]$ and $B = [-3 \quad 6]$

(ii) $A = \begin{bmatrix} 1 \\ 6 \\ 8 \end{bmatrix}$ and $B = [3 \quad 6 \quad 18]$

(iii) $A = \begin{bmatrix} 1 & 6 & 8 \\ 5 & 7 & 10 \\ 10 & 12 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -12 & 5 & 9 \\ 17 & 6 & 5 \\ 4 & 3 & 17 \end{bmatrix}$

(iv) $A = \begin{bmatrix} 7 & -11 & 7 \\ 6 & 9 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 6 \\ 8 & 7 \end{bmatrix}$.

## LONG ANSWER-I TYPE QUESTIONS

4. If A = diag (3, 4, –7) and B = diag (–8, 11, 18), then find A + B and A – B.

5. If $A = \begin{bmatrix} 2 & 3 \\ 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ 9 & 11 \end{bmatrix}$, then verify that $A + B = B + A$.

6. If $A = \begin{bmatrix} 0 & 5 \\ 7 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 5 & 9 \end{bmatrix}$ and $C = \begin{bmatrix} 11 & 8 \\ 9 & 3 \end{bmatrix}$, then verify that:

$$(A + B) + C = A + (B + C).$$

7. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

8. (i) If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, then find the matrix C such that A + B + C is a zero matrix.

(ii) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$, then find the matrix C such that A + B + C is a zero matrix.

1. If $U = [2 \quad -3 \quad 4]$, $X = [0 \quad 2 \quad 3]$, $V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then find UV + XY.

2. Show that: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

3. If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that: AB = A and BA = B.

4. For the matrices A, B and C given by

$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 5 & 7 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that:

(*i*) $A^2 = I$ (*ii*) $B^4 = O$ (*iii*) $C^2 = C$.

5. Find AB, if defined, for the following matrices:

(*i*) $A = [1 \quad 4]$, $B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ (*ii*) $A = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$

(*iii*) $A = \begin{bmatrix} 1 & 5 & 7 \\ 0 & 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 6 & 8 \\ 7 & 0 \end{bmatrix}$ (*iv*) $A = [1 \quad 2]$, $B = [4 \quad 7]$.

6. (*i*) If $[2x \quad 4]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, find the positive value of $x$.

(*ii*) Solve the following matrix equation for $x$, $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$.

(*iii*) If $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{vmatrix} x \\ 3 \end{vmatrix} = O$, find $x$.

7. If $A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}$, then show that $(A + I)(A - 4I) = O$.

8. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$.

9. Given that $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that AB = O.

10. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$.

11. If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find $A^{16}$.

## VERY SHORT ANSWER TYPE QUESTIONS

1. Find the transpose of the following matrices:

(i) $\begin{bmatrix} 1 & 6 & 8 & 9 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 8 \end{bmatrix}$

(v) $\begin{bmatrix} 5 & 6 \\ 9 & 8 \\ 11 & 18 \end{bmatrix}$ (vi) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$.

2. If A is a matrix of order $2 \times 3$ and B is a matrix of order $3 \times 5$, what is the order of matrix $(AB)'$?

3. (i) If $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, find $A + A'$.

(ii) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $A + A'$.

4. (i) If $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, find $AA'$. (ii) If $A = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}$, find $AA'$.

5. If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, find $(AB)'$.

## LONG ANSWER-I TYPE QUESTIONS

6. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 3 \\ 0 & 5 \end{bmatrix}$, verify that:

(i) $(A')' = A$ (ii) $(A + B)' = A' + B'$

(iii) $(A - B)' = A' - B'$ (iv) $(2A + 3B)' = 2A' + 3B'$.

7. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, verify that:

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$.

8. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 3 & 3 \end{bmatrix}$, then find $A^T - B^T$.

9. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.

10. (i) Find $x$, if $\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 12 & 6 \end{bmatrix}^T$.