1. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

2. Using properties of determinants, show that: $\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc\,(a-b)(b-c)(c-a)$

3. Using properties of determinants, show that: $\Delta = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

4. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a(b+c) & b(c+a) & c(a+b) \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

5. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & bc & a^2+bc \\ 1 & ca & b^2+ca \\ 1 & ab & c^2+ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

6. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = \left[(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)\right]$$

7. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & a^2 & b^3+c^3 \\ 1 & b^2 & c^3+a^3 \\ 1 & c^2 & a^3+b^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(ab+bc+ca)$$

8. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a^3 & b^3 & c^3 \end{vmatrix} = \left[-(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)\right]$$

9. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & b^2+c^2 & a^3 \\ 1 & c^2+a^2 & b^3 \\ 1 & a^2+b^2 & c^3 \end{vmatrix} = \left[-(a-b)(b-c)(c-a)(ab+bc+ca)\right]$$

1. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = \left[-(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)\right]$$

2. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} 1 & bc+ad & b^2c^2+a^2d^2 \\ 1 & ca+bd & c^2a^2+b^2d^2 \\ 1 & ab+cd & a^2b^2+c^2d^2 \end{vmatrix} = (a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$$

3. Using properties of determinants, show that: $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

4. Using properties of determinants, show that: $\begin{vmatrix} \mathrm{A}x & x^2 & 1 \\ \mathrm{B}y & y^2 & 1 \\ \mathrm{C}z & z^2 & 1 \end{vmatrix} = \begin{vmatrix} \mathrm{A} & \mathrm{B} & \mathrm{C} \\ x & y & z \\ yz & zx & xy \end{vmatrix}$

5. Using properties of determinants, show that: $\Delta = \begin{vmatrix} x+7 & x+8 & x+3a \\ x+8 & x+9 & x+3b \\ x+9 & x+10 & x+3c \end{vmatrix} = 0$.

Here a, b and c are in A.P.

6. Using properties of determinants, prove that: $\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8$.

7. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

8. Using properties of determinants, show that:

$$\Delta = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

9. Using properties of determinants, prove that:

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (x-y)(y-z)(z-x)(1+pxyz)$$